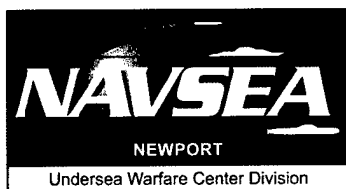


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# Calibration, Measurement, and Voltage Output of Velocity Hydrophones

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**Naval Undersea Warfare Center Division  
Newport, Rhode Island**

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## **PREFACE**

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This report discusses how to calibrate velocity hydrophones using a standard pressure hydrophone and investigates the relative contributions by the pressure and particle velocity to the measured sensitivity of such hydrophones. Sections 2 and 3 deal with the issues associated with velocity hydrophone calibration using a standard pressure hydrophone and the required correction factor for sensitivity calibration in a spherical wavefront. Section 4 summarizes the particle velocity's voltage contribution to the measured sensitivity of the hydrophone in a spherical wavefront. An example is presented in section 5 to illustrate the particle velocity's voltage contribution.

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# CALIBRATION, MEASUREMENT, AND VOLTAGE OUTPUT OF VELOCITY HYDROPHONES

## 1. INTRODUCTION

The electrical output of velocity hydrophones is proportional not only to the sound pressure but also to the particle velocity in the sound field. Therefore, the total voltage output of velocity hydrophones consists of that produced by the sound pressure and that produced by the particle velocity. Ideally, velocity hydrophones should be calibrated in terms of velocity. But, for practical reasons, these hydrophones are usually calibrated with a standard pressure hydrophone rather than a standard velocity hydrophone; hence, it is important to understand the distinction between these two voltage contributions when the acoustic sensitivity of a velocity hydrophone is calibrated in terms of pressure, especially in a spherical wavefront.

This report discusses how to calibrate velocity hydrophones using a standard pressure hydrophone and investigates the relative contributions by the pressure and particle velocity to the measured sensitivity of such hydrophones. Sections 2 and 3 deal with the issues associated with velocity hydrophone calibration using a standard pressure hydrophone and the required correction factor for sensitivity calibration in a spherical wavefront. Section 4 summarizes the particle velocity's voltage contribution to the measured hydrophone sensitivity in a spherical wavefront. An example is presented in section 5 to illustrate the particle velocity's voltage contribution.

## 2. MEASUREMENT BY VELOCITY HYDROPHONES

The measured voltage output of a velocity hydrophone ( $V_t$ ) in the sound field can be expressed as

$$V_t = V_s + V_{pv} , \quad (1)$$

where  $V_s$  is the voltage produced by the sound pressure, and  $V_{pv}$  is the voltage produced by the particle velocity. In this study, it is assumed that  $V_{pv}$  could be generated only from the particle velocity associated with the motion of pressure waves. To calibrate the acoustic sensitivity (voltage/pressure) of a velocity hydrophone, a ratio of  $V_s / \text{pressure}$  is required. Because the response of hydrophones to the particle velocity is usually unknown, the absolute value of  $V_{pv}$  cannot be obtained from the measurements unless the same type of hydrophone, with known velocity sensitivity (voltage/velocity), is used for calibration. As a result, it is not easy to separate  $V_{pv}$  from  $V_s$  in the measured voltage output  $V_t$ , and the true acoustic sensitivity

( $V_s / \text{pressure}$ ) of the velocity hydrophone cannot be determined directly from the measurements.

The particle velocity in the sound field can be specified in terms of pressure, and the relationship between the pressure and the velocity in the sound field can be derived. According to Bobber,\* the pressure  $p$  and the particle velocity  $u$  in a wave emanating from a point source can be expressed as

$$p = \left( \frac{A}{r} \right) \exp i(\omega t - kr), \quad (2)$$

$$u = \left( \frac{A}{r\rho c} \right) \left[ 1 - i \frac{\lambda}{2\pi r} \right] \exp i(\omega t - kr), \quad (3)$$

and the magnitude of the wave impedance  $\left| \frac{p}{u} \right|$  is

$$\left| \frac{p}{u} \right|^2 = (\rho c)^2 \left[ 1 + \left( \frac{\lambda}{2\pi r} \right)^2 \right], \quad (4)$$

where  $A$  is a constant,  $\lambda$  is the wavelength of the spherical wave,  $\omega$  is the frequency in rad/sec,  $t$  is time,  $k$  is the wave number,  $\rho$  is the density of the water,  $c$  is the speed of sound in the water, and  $r$  is the distance from the sound source to the hydrophones.

With a correction factor applied to the measured sensitivity ( $V_i / \text{pressure}$ ) to obtain the true acoustic sensitivity ( $V_s / \text{pressure}$ ), a velocity hydrophone, calibrated with a standard pressure hydrophone, can be used to correctly measure the pressure field.

---

\*R. J. Bobber, "Underwater Electroacoustic Measurements," Naval Research Laboratory, Washington, DC, 1970.

### 3. CALIBRATION OF VELOCITY HYDROPHONES

When the electrical output of a hydrophone is under the influence of the particle velocity in the sound field, the hydrophone sensitivity should be calibrated in terms of velocity. For practical reasons, velocity hydrophones usually are calibrated with a standard pressure hydrophone. To obtain the acoustic sensitivity ( $V_s / \text{pressure}$ ), an adjustment must be applied to the measured sensitivity ( $V_i / \text{pressure}$ ) to account for the relationship between the pressure and the velocity in the sound field.

#### 3.1 PLANE WAVES

In plane waves,  $r \gg \lambda$  and the wave impedance in equation (4) becomes a constant, as follows:

$$\left| \frac{p}{u} \right|^2 = (\rho c)^2. \quad (5)$$

As a result, the sensitivity of a velocity hydrophone expressed in terms of pressure differs from the sensitivity expressed in terms of velocity by only a constant factor  $\rho c$  in a plane wave environment. When the sensitivity is expressed in a ratio to a chosen reference value such as the one used in the decibel system, the sensitivity in terms of pressure is as accurate as the sensitivity in terms of velocity, as long as both hydrophones are in a plane wave field.

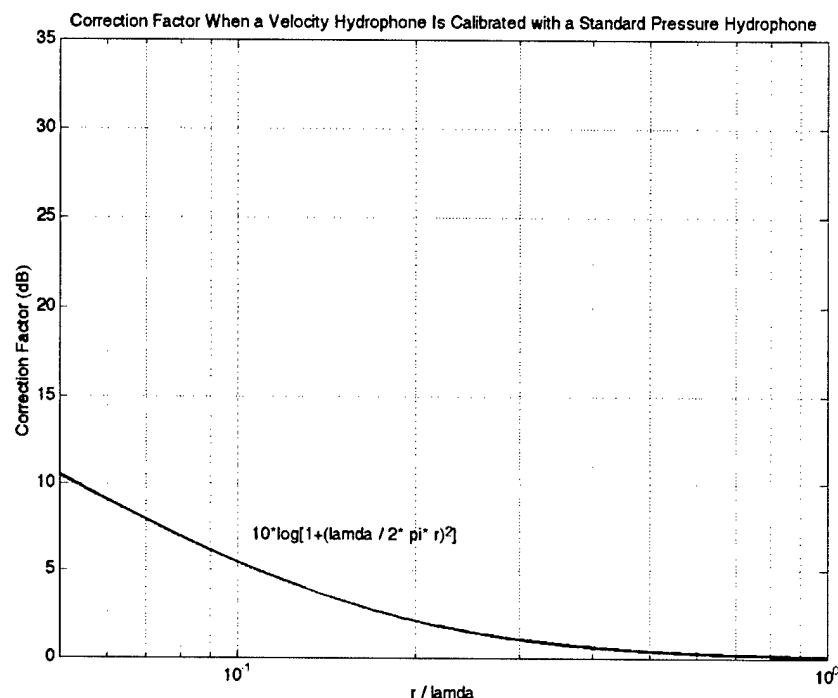
Because  $V_i / V_s$  is a constant in a plane wave environment, the ratio of measured voltage  $V_i$  of velocity hydrophones to the sound pressure measured by the standard pressure hydrophone can be directly used to calibrate the acoustic sensitivity. In other words, the voltage  $V_{pv}$  produced by the particle velocity in a plane wavefront has no effect on the accuracy of acoustic sensitivity calibration of the velocity hydrophone.

#### 3.2 SPHERICAL WAVES

In a spherical wavefront,  $p/u = \rho c$  is no longer valid. The change in wave impedance in spherical waves (equation (4)) is called velocity augmentation. Therefore, the measured sensitivity ( $V_i / \text{pressure}$ ) is subjected to a correction when the velocity hydrophone is directly calibrated with a reference to pressure. In terms of decibels, a correction factor must be subtracted from the measured sensitivity when a velocity hydrophone is calibrated with a standard pressure hydrophone in spherical waves:

$$CF = 10 * \log \left[ 1 + \left( \frac{\lambda}{2\pi r} \right)^2 \right]. \quad (6)$$

The magnitude of the correction in terms of  $r/\lambda$  is shown in figure 1 (Bobber, figure 2.38).



**Figure 1. Correction Factor in Decibels as a Function of  $r/\lambda$  in a Spherical Wavefront**

At the Naval Undersea Warfare Center's Underwater Sound Reference Detachment calibration facility in Leesburg, FL, both the standard pressure hydrophones and the velocity hydrophones are located 1.5 meters away from the sound source. These hydrophones are subjected to a spherical wave, and the wave impedance from equation (4) is used. If 1500 m/s is used for the speed of underwater sound, the correction factor to the measured sensitivity as a function of frequency  $f$  can be written as

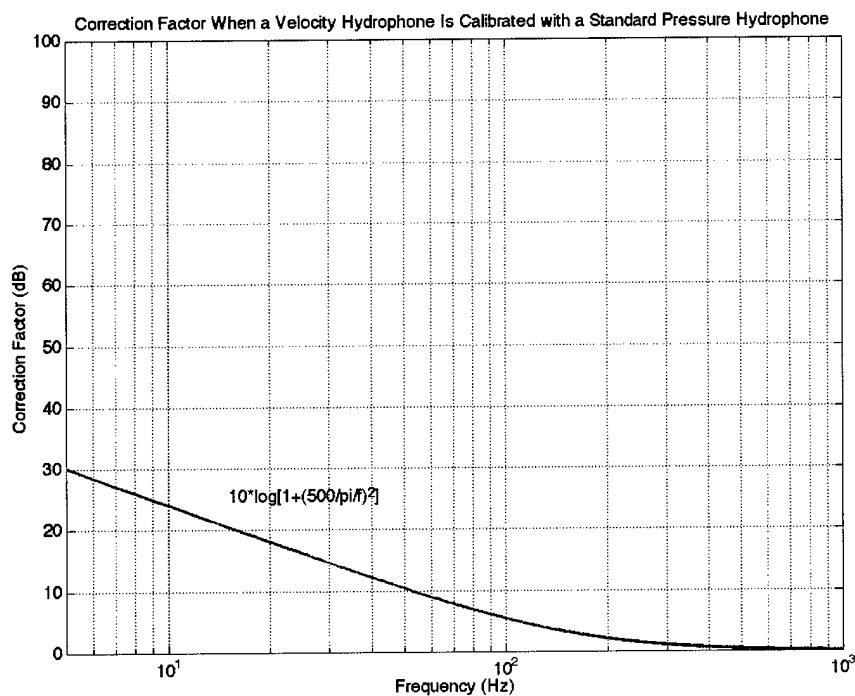
$$CF_{Leesburg} = 10 * \log \left[ 1 + \left( \frac{500 / \pi}{f} \right)^2 \right] \quad (7)$$

for velocity hydrophones.  $CF_{Leesburg}$  must be subtracted from the measured sensitivity ( $V_i / pressure$ ) for the velocity hydrophones that are calibrated at Leesburg to obtain a true measurement of acoustic sensitivity.



The magnitude of  $CF_{Leesburg}$ , expressed in decibels as a function of frequency, is shown in figure 2. It shows that the unwanted contribution to the measured sensitivity by the particle velocity is greater than 10 dB for waves with a frequency of less than 50 Hz when the velocity hydrophones were calibrated using a standard pressure hydrophone. The correction cannot be ignored for low-frequency wave measurements. On the other hand, the correction factor is less than 0.5 dB when the wave frequency is greater than 500 Hz. It means that the measured sensitivity ( $V_i / pressure$ ) of a velocity hydrophone is almost the same as the true sensitivity ( $V_s / pressure$ ) referred to pressure for wave frequencies greater than 500 Hz.

The underwater sound wavelength is 3 meters long at 500 Hz; i.e., the source-to-hydrophone distance should be greater than a half wavelength ( $r > \lambda/2$ ) if the velocity-augmenting factor is to be negligible when a velocity hydrophone is calibrated using a standard pressure hydrophone. With  $r = 1.5$  meters (as measured at the Leesburg calibration facility), this condition cannot be met for velocity hydrophones designed for low-frequency measurement ( $f < 500$  Hz). Therefore, the correction factor that arises from velocity hydrophone calibration in a spherical wavefront must be applied to the measured sensitivity ( $V_i / pressure$ ).



**Figure 2. Correction Factor in Decibels as a Function of Frequency at the Leesburg Calibration Facility**

#### 4. PARTICLE VELOCITY'S RELATIVE VOLTAGE CONTRIBUTION TO THE MEASURED SENSITIVITY

Although the absolute value of  $V_{pv}$  (or as a percentage of measured voltage  $V_t$ ) produced by the particle velocity in the sound field for a given velocity hydrophone may not be precisely known, its relative contribution to the measured sensitivity in a spherical wavefront can be deduced. In calibration with a standard pressure hydrophone, the measured voltage  $V_t$  in equation (1) can be written as

$$V_t / \text{pressure} = V_s / \text{pressure} + V_{pv} / \text{pressure}, \quad (8)$$

where  $V_t / \text{pressure}$  is the measured sensitivity,  $V_s / \text{pressure}$  is the true acoustic sensitivity, and  $V_{pv} / \text{pressure}$  is an unwanted contribution from the particle velocity due to the motion of pressure waves. As a velocity hydrophone is calibrated with a standard pressure hydrophone, the true acoustic sensitivity can be obtained using a correction factor applied to the measured sensitivity from the previous section as

$$V_s / \text{pressure} = V_t / \text{pressure} - CF, \quad (9)$$

where  $CF$  is expressed in decibels as

$$CF = 10 * \log \left( 1 + \left( \frac{\lambda}{2\pi r} \right)^2 \right). \quad (10)$$

In plane waves,  $r \gg \lambda$  results in  $CF = 0$ , which means that no matter what the value of  $V_{pv}$  is, the relative contribution from  $V_{pv}$  to the measured sensitivity expressed in the decibel system is zero. As a result, the measured sensitivity of the velocity hydrophone in terms of the pressure in planewaves could be used—without any correction—as the true acoustic sensitivity expressed in decibels.

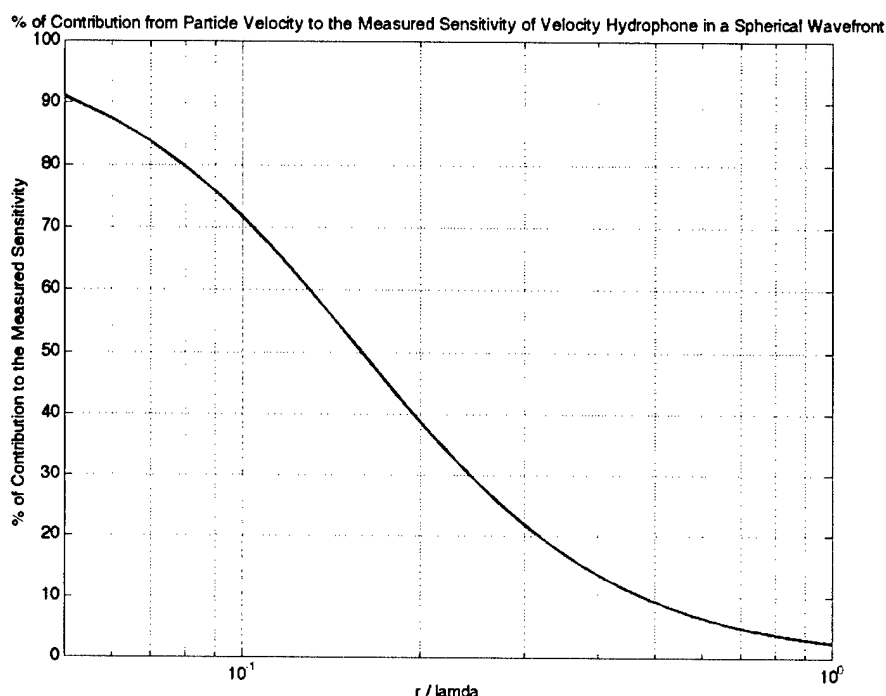
In spherical waves,  $CF$  is no longer equal to zero and the contribution from the particle velocity to the measured sensitivity cannot be ignored, especially for low-frequency waves. Expressed in decibels, equations (10) and (9) are equivalent to

$$\frac{V_s / \text{pressure}}{V_t / \text{pressure}} = \frac{1}{1 + \left( \frac{\lambda}{2\pi r} \right)^2}, \quad (11)$$

which is in the normal scale. The relative voltage contribution from the particle velocity to the measured sensitivity in a spherical wavefront, expressed in terms of percentages of the maximum contribution by the velocity hydrophone to the measured sensitivity, can be derived from equation (11) as a function of  $r/\lambda$ :

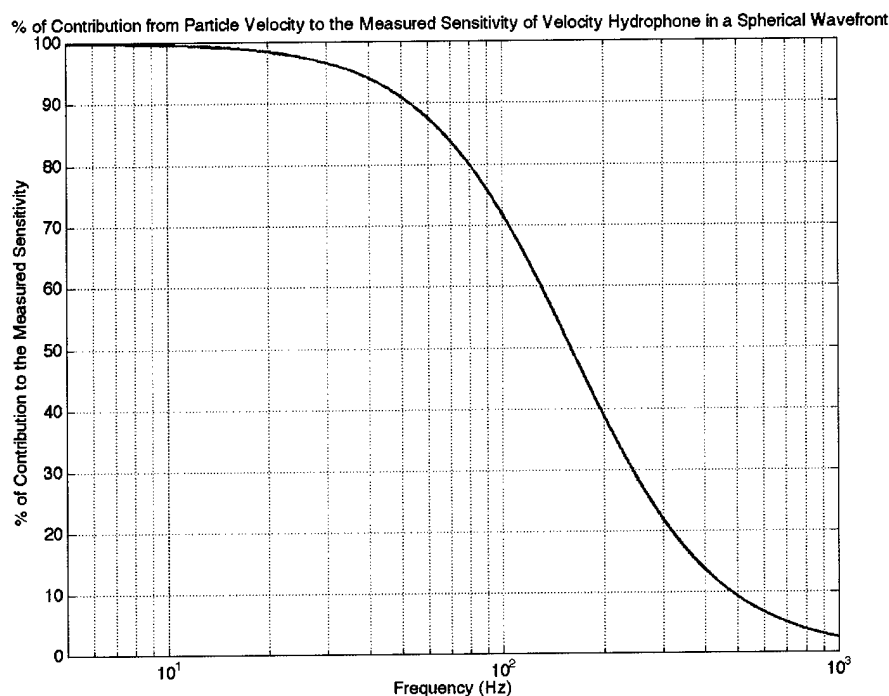
$$\frac{V_{pv}}{V_t} / \text{pressure} = \frac{V_t - V_s}{V_t} = 1 - \frac{V_s}{V_t} = 1 - \left[ \frac{1}{1 + \left( \frac{\lambda}{2\pi r} \right)^2} \right]. \quad (12)$$

Equation (12) is shown in figure 3.



**Figure 3. Percentage of Particle Velocity Contribution to the Measured Sensitivity as a Function of  $r/\lambda$  in a Spherical Wave**

For velocity hydrophones calibrated in terms of pressure at the Leesburg facility with hydrophone-to-sound source distance of  $r = 1.5$  meters, the relative particle velocity voltage contribution to the measured sensitivity in a spherical wavefront can be plotted as a function of frequency, as shown in figure 4. It is clear that the relative voltage contribution from the particle velocity of wave motion to the measured sensitivity is greatest for a low-frequency wave (long wavelength). Its relative contribution to the velocity hydrophones calibrated at Leesburg drops below 10% for waves with a frequency greater than 500 Hz.



**Figure 4. Percentage of Particle Velocity Contribution to the Measured Sensitivity as a Function of Frequency Measured at the Leesburg Calibration Facility**

## 5. EXAMPLE PARTICLE VELOCITY'S VOLTAGE CONTRIBUTION

The sensitivity of velocity hydrophones can be calibrated with a standard pressure hydrophone using a correction factor that accounts for the velocity augmentation in a spherical wave. With the "correct" acoustic sensitivity, velocity hydrophones can be used to measure the pressure field in exactly the same way as traditional hydrophones. In this regard, the value of the voltage produced by the particle velocity of wave motion has no direct effect on the pressure measurements in a plane or spherical wave.

The question now is: Can the voltage produced by the particle velocity and the voltage produced by the pressure in measurements taken using a velocity hydrophone be separated, as shown in equation (1)? Various velocity hydrophones respond differently to the particle velocity impinging on them differently. Without knowing exactly how a velocity hydrophone will respond to the particle velocity, the voltage ( $V_{pv}$ ) produced by the particle velocity cannot be known. However, a partial answer to this question can be found if a velocity hydrophone has been calibrated in terms of velocity and has known voltage contribution "characteristics" by the particle velocity.

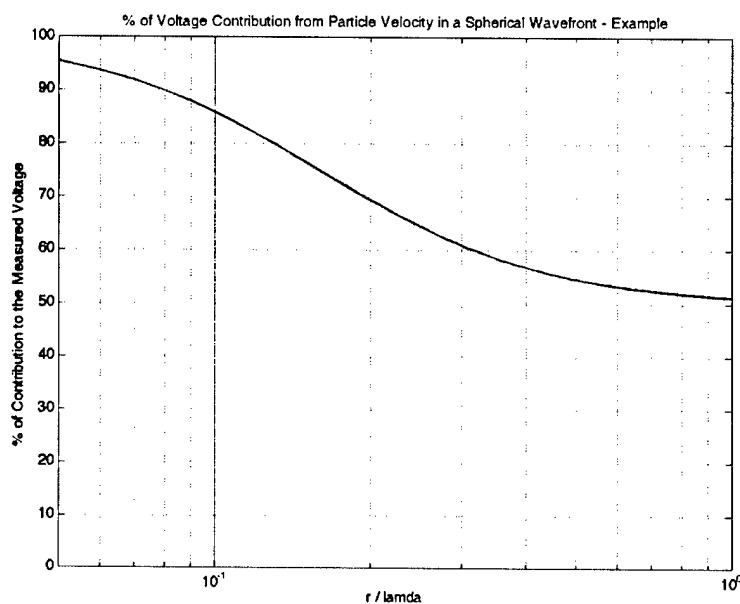
Consider the following example: A velocity hydrophone produced 50% of the voltage output by the particle velocity in plane waves. (This velocity hydrophone can be calibrated by

using the procedures outlined in this report.) Assume that the velocity hydrophone has a flat voltage response to the pressure impinging on it. This means that  $V_s$ , which is the voltage produced by the sound pressure, has a constant value for the frequency range of interest. The voltage  $V_{pv}$  produced by the particle velocity in terms of percentage of the measured voltage can be derived from equation (12) as

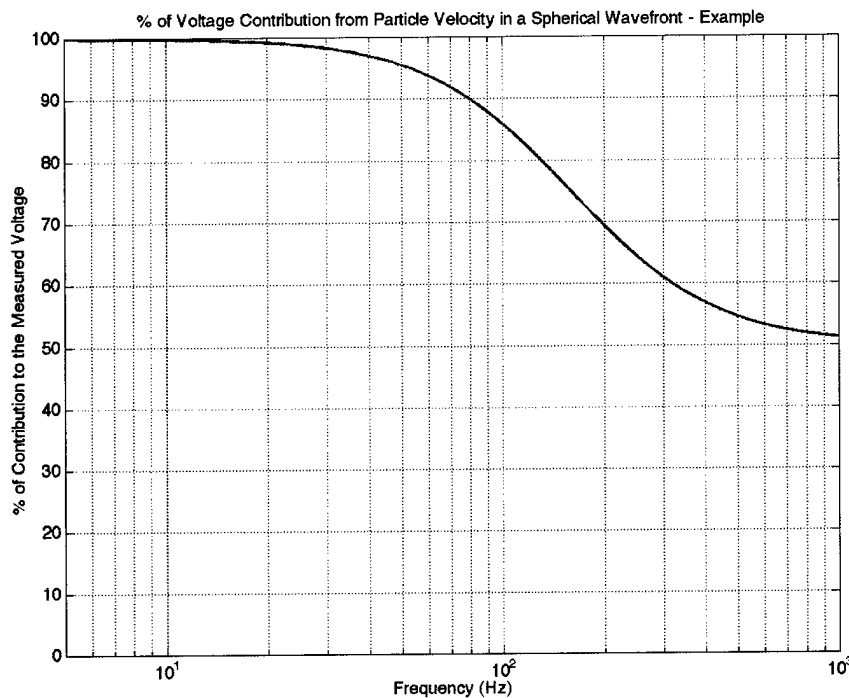
$$\frac{V_{pv}}{V_t} = 1 - \left[ 0.5 * \frac{1}{1 + \left( \frac{\lambda}{2\pi r} \right)^2} \right]. \quad (13)$$

Figure 5 shows the particle velocity contribution to the measured voltage as a function of  $r/\lambda$  in a spherical wavefront. The particle motion produced 50% of the measured voltage at very high frequency, as specified in the example for plane waves. The contribution by the particle motion to the measured voltage increases for a lower frequency wave. At very low frequency, the particle motion produced most of the measured voltage.

If this velocity hydrophone had been calibrated at the Leesburg facility with a hydrophone-to-sound-source distance of  $r = 1.5$  meters, the voltage contribution by the particle motion could be plotted as a function of frequency (figure 6). For waves with frequencies of less than 80 Hz, the contribution by the particle motion to the measured voltage is greater than 90%. For  $r \gg \lambda$  or high-frequency waves, the pressure wave reaches the limit of a plane wave and the voltage produced by the particle velocity is about 50% of the measured voltage, which is specified as a known limit in the example above. This is also shown in figures 5 and 6.



**Figure 5. Percentage of Particle Velocity Contribution to the Measured Voltage as a Function of  $r/\lambda$  in a Spherical Wave**



***Figure 6. Percentage of Particle Velocity Contribution to the Measured Voltage as a Function of Frequency Measured by a Leesburg-Facility-Calibrated Velocity Hydrophone***

## 6. CONCLUSIONS

The following conclusions may be drawn regarding velocity hydrophone calibration:

1. Because of velocity augmentation in a spherical wave emanating from a point sound source, to obtain the true acoustic sensitivity a correction factor must be subtracted from the measured sensitivity when a velocity hydrophone is calibrated using a standard pressure hydrophone.
2. Using a standard pressure hydrophone, the magnitude of the correction factor for the velocity hydrophones calibrated in the Leesburg facility is greater than 10 dB for waves with frequencies 50 Hz or lower. The correction factor is less than 0.5 dB when the wave frequency is greater than 500 Hz.
3. The correction factor for velocity hydrophone calibration can be neglected only if the source-to-hydrophone distance is greater than a half wavelength. This is true even in the spherical wave environment. In the Leesburg calibration facility, with  $r = 1.5$  meters, this corresponds to sound waves with frequencies that are greater than 500 Hz.

4. With a "correct" frequency calibration, any velocity hydrophone can be used to measure the pressure or noise field in exactly the same manner as that of a traditional pressure hydrophone.

5. Although the exact values of the voltage produced by the particle velocity in velocity hydrophones are not usually known from the measurements, the relative contribution of the particle motion to the measured sensitivity can be deduced. In the plane wave environment with  $p/u = \rho c$ , the voltage output produced by the particle velocity is a constant percentage of the measured voltage at all frequencies for a given velocity hydrophone. Its contribution to the measured acoustic sensitivity, expressed in decibels, is zero. In the spherical environment, the relative voltage contribution of the particle velocity to the measured sensitivity is a function of  $r/\lambda$  or a function of frequency when  $r$  is known. The results are shown in figure 3 for general calibration and in figure 4 for the Leesburg facility calibration.

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